I

A Summary of the Evans Field Theory

I.1 The Homogeneous Field Equation

Barebones Notation

$$D \wedge F = R \wedge A \tag{I.1}$$

Tangent Bundle Notation

$$D \wedge F^a = R^a{}_b \wedge A^b \tag{I.2}$$

Complete Index Notation

$$(D \wedge F^a)_{\mu\nu\rho} = \left(R^a{}_b \wedge A^b\right)_{\mu\nu\rho} \tag{I.3}$$

Spin Connection Notation

$$\left(d \wedge F^a + \omega^a{}_b \wedge F^b\right)_{\mu\nu\rho} = \left(R^a{}_b \wedge A^b\right)_{\mu\nu\rho} \tag{I.4}$$

Tensor Notation

$$\begin{aligned}(d \wedge F)^a_{\mu\nu\rho} &= \partial_\mu F^a{}_{\nu\rho} + \partial_\nu F^a{}_{\rho\mu} + \partial_\rho F^a{}_{\mu\nu} \\ (\omega \wedge F)^a{}_{\mu\nu\rho} &= \omega^a{}_{\mu b} F^b{}_{\nu\rho} + \omega^a{}_{\nu b} F^b{}_{\rho\mu} + \omega^a{}_{\rho b} F^b{}_{\mu\nu} \\ (R \wedge A)^a{}_{\mu\nu\rho} &= R^a{}_{b\mu\nu} A^b{}_\rho + R^a{}_{b\nu\rho} A^b{}_\mu + R^a{}_{b\rho\mu} A^b{}_\nu\end{aligned} \tag{I.5}$$

Eq.(I.5) give the most complete expression of the homogeneous field equation, i.e.:

$$\begin{gathered}\partial_\mu F^a{}_{\nu\rho} + \partial_\nu F^a{}_{\rho\mu} + \partial_\rho F^a{}_{\mu\nu} + \omega^a{}_{\mu b} F^b{}_{\nu\rho} + \omega^a{}_{\nu b} F^b{}_{\rho\mu} + \omega^a{}_{\rho b} F^b{}_{\mu\nu} = \\ R^a{}_{b\mu\nu} A^b{}_\rho + R^a{}_{b\nu\rho} A^b{}_\mu + R^a{}_{b\rho\mu} A^b{}_\nu\end{gathered} \tag{I.6}$$

Maxwell-Heaviside Limit

$$\begin{gathered}R^a{}_b \wedge A^b = \omega^a{}_b \wedge F^b \\ \partial_\mu F^a{}_{\nu\rho} + \partial_\nu F^a{}_{\rho\mu} + \partial_\rho F^a{}_{\mu\nu} = 0\end{gathered} \tag{I.7}$$

The tangent bundle is not identified, so:

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0 \tag{I.8}$$

Eq.(I.8) is the homogeneous field equation of the MH theory. Eq. (I.8) cannot describe the effect of gravitation on electromagnetism and vice-versa, because it is a flat spacetime equation of special relativity.

I.2 Hodge Dual of the Homogeneous Field Equation

The Hodge dual of the homogenous field equation is a re-expression of the Bianchi identity and therefore contains the same information. In the general 4-D manifold the Hodge dual must be precisely defined as follows.

The general definition of the Hodge dual is given by Carroll.

$$\widetilde{\mathcal{X}}_{\mu_1\ldots\mu_{n-p}} = \frac{1}{p!}\epsilon^{\nu_1\ldots\nu_p}{}_{\mu_1\ldots\mu_{n-p}}\mathcal{X}_{\nu_1\ldots\nu_p} \tag{I.9}$$

In a general n-dimensional manifold Eq.(I.9) maps from a p-form of differential geometry to an (n-p)-form. The general Levi-Civita symbol is defined in any manifold to be:

$$\epsilon_{\mu_1\mu_2\ldots\mu_n} = \begin{cases} 1 \text{ if } \mu_1\mu_2\ldots\mu_n \text{ is an even permutation} \\ -1 \text{ if } \mu_1\mu_2\ldots\mu_n \text{ is an odd permutation} \\ 0 \qquad\qquad \text{otherwise} \end{cases} \tag{I.10}$$

The Levi-Civita tensor used in the definition of the Hodge dual, is:

$$\epsilon_{\mu_1\mu_2\ldots\mu_n} = (|g|)^{1/2}\,\widetilde{\epsilon}_{\mu_1\mu_2\ldots\mu_n} \tag{I.11}$$

where $|g|$ is the numerical magnitude of the determinant of the metric. In a four-dimensional manifold a two-form is dual to a two-form:

$$\widetilde{\mathcal{X}}_{\mu_1\mu_2} = \frac{1}{2}\epsilon^{\nu_1\nu_2}{}_{\mu_1\mu_2}\mathcal{X}_{\nu_1\nu_2} \tag{I.12}$$

Indices are raised and lowered on the Levi-Civita tensor by use of the metric tensor. The latter is normalized by:

$$g^{\mu\nu}g_{\mu\nu} = 4 \tag{I.13}$$

Therefore we have results such as:

$$\widetilde{\mathcal{X}}_{\mu\nu} = \frac{1}{2}\epsilon^{\rho\sigma}{}_{\mu\nu}\mathcal{X}_{\rho\sigma} \tag{I.14}$$

with, for example:

$$\epsilon_{\sigma\mu\nu\rho} = g_{\sigma\kappa}\epsilon^{\kappa}{}_{\mu\nu\rho} \tag{I.15}$$

We may then rewrite eqn.(I.4):

$$\boxed{\partial^{\mu}\widetilde{F}^{a}{}_{\mu\nu} = \mu_0 \widetilde{j}^{a}{}_{\nu}} \tag{I.16}$$

Note that eqn.(I.4) and (I.16) contain the same information, they are both expressions of the homogeneous field equation of objective or generally covariant physics.

I.3 The Inhomogeneous Field Equation

The homogeneous field equations eqn.(I.4) and (I.16) is the generally covariant form of Gauss Law applied to magnetism and the Faraday Law of induction. The inhomogeneous field equation is deduced from eqn. (I.1) as follows:

$$\boxed{D \wedge \widetilde{F} = \widetilde{R} \wedge A} \tag{I.17}$$

The complete description is therefore:

$$\boxed{\begin{aligned} d \wedge F &= R \wedge A - \omega \wedge F = \mu_0 j \\ d \wedge \widetilde{F} &= \widetilde{R} \wedge A - \omega \wedge \widetilde{F} = \mu_0 J \end{aligned}} \tag{I.18}$$

eqn.(I.18) are the generally covariant forms of the four fundamental laws of electromagnetics.

In order to CAD/CAM a circuit working from the general 4-D manifold known as "Evans Spacetime" eqn.(I.18) must be solved simultaneously. The mathematical problem is one of solving simultaneous partial differential tensor equations with given initial and boundary conditions.

I.3.1 The Standard Model

The equivalents of eqn.(I.18) is the standard model are:

$$\boxed{\begin{aligned} d \wedge F &= 0 \\ d \wedge \widetilde{F} &= \mu_0 J \end{aligned}} \tag{I.19}$$

Eqs (I.19) are equations of a flat or Minkowski spacetime. Eqs (I.19) are not objective equations of physics because they are not equations of general relativity. There is no indication in eqs (I.19) that J is derived from the wedge product of the Riemann and tetrad forms. The Evans field equations (I.18) show that

$$\boxed{J = \frac{A^{(0)}}{\mu_0}\left(\widetilde{R} \wedge q - \omega \wedge \widetilde{T}\right)} \tag{I.20}$$

Eqn. (I.20) shows that Evans spacetime is a source of electric charge/current density.

It is therefore very important to evaluate J numerically. For a given potential:

$$A = A^{(0)} q \tag{I.21}$$

and given curvature:

$$R = D \wedge \omega \tag{I.22}$$

we need to calculate J. In eqn. (I.22) ω is the spin connection, related to the Christoffel connection.

I.4 Summary of the Unified Field Theory

Any situation in field theory is described by:

$$D \wedge F^a = R^a{}_b \wedge A^b \tag{I.23}$$

$$D \wedge \widetilde{F}^a = \widetilde{R}^a{}_b \wedge A^b \tag{I.24}$$

where:

$$F^a = A^{(0)} T^a \tag{I.25}$$

$$A^a = A^{(0)} q^a \tag{I.26}$$

and:

$$D \wedge F^a = d \wedge F^a + \omega^a{}_b \wedge F^b \tag{I.27}$$

the fundamental charge-current three-forms are defined by:

$$j^a = \frac{1}{\mu_0}\left(R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b\right) \tag{I.28}$$

$$J^a = \frac{1}{\mu_0}\left(\widetilde{R}^a{}_b \wedge A^b - \omega^a{}_b \wedge \widetilde{F}^b\right) \tag{I.29}$$

In eqs (I.23–I.29):

$$
\begin{aligned}
A^{(0)} &= \text{Fundamental potential mangnitude in volts} \\
q^a &= \text{vector valued tetrad one-form} \\
T^a &= \text{vector valued torsion two-form} \\
R^a{}_b &= \text{tensor valued curvature two-form} \\
\omega^a{}_b &= \text{spin-connection one-form} \\
F^a &= \text{vector valued electromagnetic field two-form} \\
A^a &= \text{vector valued electromagnetic potential one-form} \\
j^a &= \text{homogeneous current three-form} \\
J^a &= \text{inhomogeneous current three-form} \\
d\wedge &= \text{exterior derivative} \\
D\wedge &= \text{covariant exterior derivative} \\
\mu_0 &= \text{permeability is vacuo}\,(SI)
\end{aligned}
$$

The equations below are in SI units. $\widetilde{F}^a$ is the hodge dual of F^a in the general 4-d manifold (Evans spacetime) and $\widetilde{R}^a{}_b$ is the Hodge dual of $R^a{}_b$ in Evans spacetime.

The following are well-known limiting forms of the Evans field theory.

I.5 Einstein Field Theory of Gravitation

This limit is defined by:

$$F^a = 0 \tag{I.30}$$

$$R^a{}_b \wedge A^b = 0 \tag{I.31}$$

$$\widetilde{F}^a = 0 \tag{I.32}$$

$$\widetilde{R}^a{}_b \wedge A^b \neq 0 \tag{I.33}$$

Eqn.(I.31) implies that the Christoffel symbol is symmetric in its lower two indices. This self-consistantly implies:

$$T^a(\text{Einstein}) = 0 \tag{I.34}$$

Self-consistantly therefore, in the Einstein field theory of gravitation, there is no electromagnetic field present. In this theory the torsion tensor is zero because it is defined as the difference:

$$T^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\mu\nu} - \Gamma^\kappa{}_{\nu\mu} = 0 \tag{I.35}$$

Metrics that obey conditions (I.35) cannot be used in a unified field theory, they can only be used to describe gravitation.

I.6 Maxwell-Heaviside Field Theory of Electromagnetism

This is the limit described by:

$$d \wedge F^a = 0 \tag{I.36}$$

$$d \wedge \widetilde{F}^a = \mu_0 J^a \tag{I.37}$$

Therefore:

$$R^a{}_b \wedge A^b = \omega^a{}_b \wedge F^b \tag{I.38}$$

$$j^a = 0 \tag{I.39}$$

$$\widetilde{R}^a{}_b \wedge A^b \neq \omega^a{}_b \wedge \widetilde{F}^b \tag{I.40}$$

$$J^a \neq 0 \tag{I.41}$$

The Evans spacetime reduces to Minkowski spacetime, so $D\wedge$ is replaced by $d\wedge$. In the original nineteenth century Maxwell Heaviside theory, the field is an entity superimposed on a flat Euclidean 3-D space and the concept of time is distinct from that of space. The existence of the tangent bundle index a is not recognized, neither is that of the spin connection $\omega^a{}_b$ and curvature $R^a{}_b$. The current J is essentially empirical in MH field Theory.